

Head-on Collision of Two Unequal Mass Black Holes

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We present results from the first fully nonlinear numerical calculations of the head-on collision of two unequal mass black holes. Selected waveforms of the most dominant $\ell = 2, 3$ and 4 quasinormal modes are shown, as are the total radiated energies and recoil velocities for a range of mass ratios and initial separations. Our results validate the close and distant separation limit perturbation studies, and suggest that the head-on collision scenario is not likely to produce an astrophysically significant recoil effect.

The collision of two black holes is expected to be one of the most interesting and important astrophysical sources of gravitational radiation. The strong bursts of radiation emitted from the interaction should be detectable by the next generation gravitational wave observatories, and will provide explicit evidence for the existence of black holes, assuming the characteristic signatures of the holes are observed. To compliment the anticipated observations, several calculations of binary black hole interactions have been performed in recent years, inspired in part by a broader Grand Challenge effort to supply waveform templates that are essential for the analysis of observational data. These calculations have been based on axisymmetric numerical relativity computations of the head-on collision of two initially time-symmetric and boosted black holes [1,2], as well as perturbation studies [2,3] which confirm and elucidate the numerical results in the close limit approximation (CLAP). However, the fully nonlinear numerical studies performed until now have been restricted to the simplified case of equal mass black hole systems. To extend these calculations, we have recently introduced a new class of body-fitting coordinate systems (class I in the notation of Ref. [4]) that overcomes the coordinate singularity problems in previous work. In this paper we utilize these new coordinates to carry out the first numerical studies of axisymmetric head-on collisions of unequal mass black holes. In addition to expanding the catalog of computed energies and waveforms, numerical investigations of these systems can address whether gravitational radiation from the collisions is likely to carry an astrophysically significant component of linear momentum, and whether substantial recoil velocities can be imparted to the final black hole.

The initial data used in our simulations are time-symmetric analytic solutions that generalize the equal mass Misner [5] data adopted in previous numerical calculations [1]. Applying the isometry operation systematically across each of the throats for the infinite image poles, the conformal factor can be written in cylindrical coordinates as

$$\Psi = 1 + \sum_{n=1}^{\infty} \left[\frac{\text{csch} \mu_{n,1}}{\sqrt{\rho^2 + (z - \coth \mu_{n,1})^2}} + \frac{\text{csch} \mu_{n,2}}{\sqrt{\rho^2 + (z + \coth \mu_{n,2})^2}} \right], \quad (1)$$

where we have defined $\mu_{n,1} = \mu_{n,2} = n\mu_0$ for even n , $\mu_{n,1} = \mu_1 + (n-1)\mu_0$ and $\mu_{n,2} = \mu_2 + (n-1)\mu_0$ for odd n , and $\mu_0 = (\mu_1 + \mu_2)/2$. The initial data is thus uniquely specified by two parameters, μ_1 and μ_2 , which determine the black hole masses and separations, analogous to the single Misner parameter in the equal mass case. The physical content of the initial data is characterized by the total (ADM) mass of the spacetime derived as the asymptotic leading order term in $M_{ADM} = -2r^2 \partial_r \Psi$

$$M_{ADM} = 2 \sum_{n=1}^{\infty} [\text{csch}(\mu_{n,1}) + \text{csch}(\mu_{n,2})], \quad (2)$$

the proper distance separating the black hole throats along the ($\rho = 0$) axis

$$L = 2 + \frac{2\mu_2}{\sinh \mu_2} + 2 \sum_{n=1}^{\infty} \left[\frac{4n\mu_0}{\sinh 2n\mu_0} + \frac{2n\mu_0 - \mu_2}{\sinh(2n\mu_0 - \mu_2)} + \frac{2n\mu_0 + \mu_2}{\sinh(2n\mu_0 + \mu_2)} \right], \quad (3)$$

and the bare masses of the two holes as suggested by Lindquist [6,7]

$$m_1 = 2 \sum_{n=1}^{\infty} \left[\frac{2n}{\sinh 2n\mu_0} + \frac{n}{\sinh(2n\mu_0 - \mu_2)} + \frac{n}{\sinh(2n\mu_0 + \mu_2)} \right], \quad (4)$$

$$m_2 = 2 \sum_{n=1}^{\infty} \left[\frac{2n}{\sinh 2n\mu_0} + \frac{n}{\sinh(2n\mu_0 - \mu_1)} + \frac{n}{\sinh(2n\mu_0 + \mu_1)} \right]. \quad (5)$$

The numerical solutions are parameterized by the ratio of bare masses $M_R = m_1/m_2$ with the convention $m_1 \leq m_2$, and the proper separation between the throats L/M normalized to the maximum of the two masses. Here we simply define $M = M_{ADM}/(1 + M_R)$ for consistency with previous work. The code is applied to collide black holes with initial separations up to $\sim 10M$ with mass ratios $M_R = 1, 0.75, 0.5$ and 0.25 . These choices are determined mainly by stability issues, which require $(\mu_1, \mu_2) \lesssim 3.0$ as the upper bound [4], and accuracy issues which require $(\mu_1, \mu_2) \gtrsim 0.5$ to resolve the very weak higher order ($\ell \geq 3$) multipole waveforms, although an even greater range of parameters can be evolved accurately in the stronger $\ell = 2$ signals. For each of the cases presented, the code is stable enough to maintain accurate solutions to well past $100M_{ADM}$, which is more than enough time to monitor the complete wave signals with current parameters. Although results are presented for $M_R \geq 0.25$, we expect that this range of parameters will produce maximal or near maximal recoil velocities, a conclusion supported by the CLAP [7] and Fitchett [8] calculations which predict maximum recoil at $M_R \sim 0.25$ and 0.38 respectively.

The method we use to compute waveforms is based on the technique developed by Abrahams and Evans [9] (and applied in Ref. [10]) to extract the Regge–Wheeler perturbation functions and to construct the gauge invariant Zerilli function ψ . Fig. 1 shows a characteristic sample of generated waveforms for the particular case of $\mu_1 = 2.5$ and $\mu_2 = 2$ corresponding to $M_R = 0.66$ and $L/M = 7.45$. Also shown are the quasinormal mode fits from perturbation theory using a linear combination of the fundamental and first overtone for the final black hole of mass M_{ADM} . The fits nicely match both the frequency and damping rate in the late-stage ringing phase. In Fig. 2 we show the total energy radiated in the dominant $\ell = 2$ mode as a function of initial separation distance for different mass ratios. For comparison, we note that the perturbation results of Davis et al. (DRPP) [11], when rescaled by the reduced mass, yields $E/M_{ADM} \approx 0.0104\mu^2/M_{ADM}^2 \approx 0.0104M_R^2/(1 + M_R)^3$ for the radiated energy in the large separation limit, where μ is the reduced mass. Our numerical results are in remarkable agreement with this prediction, which suggests that the radiated energy (when normalized to the ADM mass) decreases as M_R^2 and yields maximum energy emissions of $(3, 7, 10$ and $13) \times 10^{-4}$ for the $M_R = (0.25, 0.5, 0.75$ and $1.0)$ cases respectively. In the opposite close separation limit, our results are also in excellent agreement with the CLAP calculations of Andrade and Price [7], and the particle limit ($M_R \rightarrow 0$) collisions from finite initial separations based on the Green’s function integration by Lousto and Price [12]. This last curve appropriately bounds the sequence of data points from our numerical relativity calculations.

As indicated by Fig. 1, the quasinormal modes are strongly excited, and the black hole oscillations are well described by essentially the quasinormal modes (and particularly the $\ell = 2$ mode) of the final black hole despite the fact that the systems considered are not small perturbations. In this regard, the collision of unequal mass black holes is similar to that of equal mass systems, except now the breaking of equatorial symmetry allows for odd mode multipole components. The mixing of adjacent multipole modes gives rise to a nonvanishing flux of linear momentum along the z -axis which can be evaluated using the Landau–Lifshitz pseudotensor [13,14]

$$\frac{dP^z}{dt} = \frac{1}{32\pi r^2} \int_0^\pi \left(\frac{\partial g_{\theta\theta}}{\partial t} - \frac{\partial g_{\phi\phi}}{\partial t} \frac{1}{\sin^2 \theta} \right)^2 \cos \theta \sin \theta d\theta. \quad (6)$$

As an additional check we have also computed the momentum flux from consecutive Zerilli components [7]

$$\frac{dP^z}{dt} = \frac{1}{16\pi} \sum_{\ell=2}^{\infty} \sqrt{\frac{(\ell-1)(\ell+3)}{(2\ell+1)(2\ell+3)}} \frac{d\psi_\ell}{dt} \frac{d\psi_{\ell+1}}{dt}, \quad (7)$$

and we find generally good agreement (typically 10 - 30% relative differences) between the two calculations. As a result of the momentum emission from odd/even mode mixing, the final black hole will acquire a recoil velocity $v^z = -M_{ADM}^{-1} \int (dP^z/dt) dt$, opposite in direction to the momentum flux of the waves.

The radiation of momentum by gravitational waves is of interest in work relating to active galactic nuclei, quasars and even archetypical galaxies since highly dynamic black holes and coalescing binary black hole systems may abound in galactic discs as well as in the centers of galactic nuclei. Because the efficiency of momentum radiation emission is not known precisely, the stability of these systems remains in question due to the generation of a recoil velocity in the affected black holes, which, for sufficiently asymmetric configurations, may be large enough to break black holes free from the host galaxy. If so, gravitational radiation effects will have considerable observable consequences for astrophysics and cosmology (such as the depletion of black holes from host galaxies, the disruption of active galactic core energetics, and the introduction of black holes and stellar material into the IGM) since the processes generating the momentum emission, i.e., stellar core collapse and binary mergers, are likely to be fairly common.

Although fully nonlinear numerical simulations of asymmetric black hole systems have not been performed to now, several quasi-Newtonian and perturbation calculations have been carried out over the past two decades to estimate the magnitude of this recoil effect. For example, Bekenstein [15] derived an upper bound of $v_r \leq 300$ km/sec for the recoil velocity from nonspherical stellar core collapse; Moncrief [16] found $v_r = 25$ km/sec for small

non-spherical distortions in Oppenheimer-Snyder collapse models; and Nakamura and Haugan (NH) [17] computed $v_r = |\Delta p|/M \sim 262(m_1/M)^2$ km/sec for a test particle of mass m_1 plunging from rest at infinity into a Schwarzschild black hole of mass M , with $m_1 \ll M$. Extrapolating the NH result to the comparable mass limit ($m_1 \sim M$), we replace the mass dependency with Fitchett's [8] scaling formula, $(m_1/M)^2 \rightarrow f(M_R) = M_R^2(1-M_R)(1+M_R)^{-5}$, which has the desired perturbation and equal mass limits. Considering the maximal value of $f = 0.01789$ at $M_R = 0.38$, the perturbation result predicts a maximum recoil velocity of about 5 km/sec for the head-on collision scenario.

However, numerical calculations are needed to adequately resolve the equal or near-equal mass cases, and to properly model nonlinear effects from internal dynamics such as tidal heating, and the re-absorption and beaming of gravitational waves. We plot in Fig. 3 the absolute value of the recoil velocities found in our simulations by evaluating the integrated momentum flux from the Landau-Lifshitz pseudotensor (6) across a 2-surface of radius $15M_{ADM}$. Despite the fact that we are unable to accurately evolve black hole collisions with initial separations greater than about $10M$, Fig. 3 does suggest a maximum recoil on the order of $v_r \sim 10 - 20$ km/sec, in rough agreement with the various perturbation and quasi-Newtonian estimates. At sufficiently small initial separations, our results also agree nicely with CLAP, especially when considering the smallness of this effect and the estimates of numerical errors based on resolution studies (less than 10% variation between fixed detectors in the 200 and 300 radial zone evolutions) and extractions of the momentum across differently positioned detectors (roughly 10-30% variation between detectors at radii 15 and $25 M_{ADM}$). In comparison, the escape velocity from galactic structures with radius R and total mass M is of order $\sqrt{GM/R}$, which can vary from anywhere between about 200 km/sec for the less massive compact dwarf galaxies to about 1000 km/sec for giant ellipticals (though black holes in the galactic disk and far from the core would require substantially less recoil in the direction of galactic rotation to achieve escape velocity). We thus conclude that the head-on collision of black holes is not likely to produce an astrophysically significant recoil effect.

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- [1] P. Anninos, D. Hobill, E. Seidel, L. Smarr, and W.-M. Suen, Phys. Rev. Lett. 71, 2851 (1993)
 - [2] Baker, J., Abrahams, A., Anninos, P., Brandt, S., Price, R., Pullin, J. and Seidel, E., Phys. Rev. D, 55, 829 (1997)
 - [3] P. Anninos, R.H. Price, J. Pullin, E. Seidel, W.M. Suen, Phys. Rev. D 52, 4462 (1995)
 - [4] P. Anninos, S.R. Brandt and P. Walker, gr-qc/9712047
 - [5] C. Misner, Phys. Rev. D. 118, 1110 (1960)
 - [6] R.W. Lindquist, J. Math. Phys. 4, 938 (1963)
 - [7] Z. Andrade and R.H. Price, gr-qc/9611022
 - [8] M.J. Fitchett, MNRAS 203, 1049 (1983)
 - [9] A. Abrahams and C. Evans, Phys. Rev. D 42, 2585 (1990)
 - [10] A. Abrahams, D. Bernstein, D. Hobill, E. Seidel and L. Smarr, Phys. Rev. D 45, 3544 (1992)
 - [11] M. Davis, R. Ruffini, H. Press, and R. H. Price, Phys. Rev. Lett. 27, 1466 (1971).
 - [12] C. Lousto and R.H. Price, Phys. Rev. D. 55, 2124-2138 (1997) gr-qc/9609012
 - [13] Landau, L.D. and Lifshitz, E.M., *The Classical Theory of Fields*, (Pergamon Press, 1975)
 - [14] Cunningham, C.T., Price, R.H. and Moncrief, V., Astrophys. J. 230, 870 (1979)
 - [15] J.D. Bekenstein, Astrophys. J. 183, 657 (1973)
 - [16] V. Moncrief, Astrophys. J. 238, 333 (1980)
 - [17] T. Nakamura and M.P. Haugan, Astrophys. J. 269, 292 (1983)

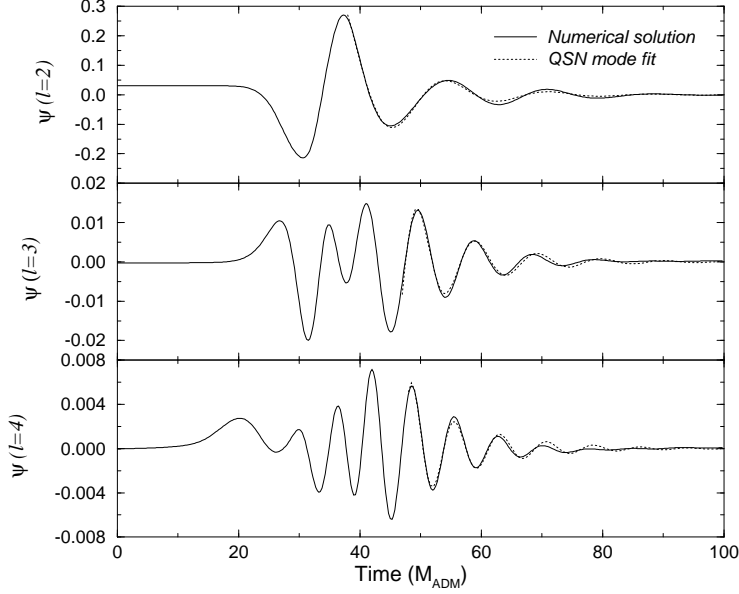


FIG. 1.

Zerilli waveforms for the $\mu_1 = 2.5$ and $\mu_2 = 2$ case, corresponding to a mass ratio $M_R = m_1/m_2 = 0.66$ and proper initial separation $L/M = 7.45$. The $\ell = 2, 3$, and 4 modes are displayed normalized to the ADM mass, and extracted at a radius of $20M_{ADM}$. Also shown with dotted lines are fits to the fundamental and first overtone of the quasinormal modes from perturbation theory. The simulations are run at a grid resolution of 200×70 (radial \times angular) zones.

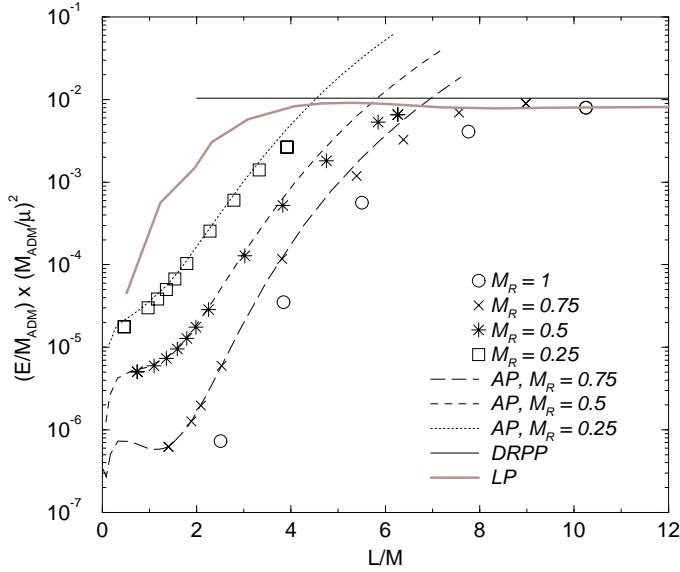


FIG. 2.

The total energy radiated in the dominant $\ell = 2$ mode from the head-on collision of two black holes as a function of separation distance L/M between the holes, where $M = M_{ADM}/(1 + M_R)$ is the approximate mass of the larger black hole, and $M_R = m_1/m_2$ is the ratio of bare masses. The energies are computed as the integrated flux across a 2-surface of radius $15M_{ADM}$, and rescaled by the ADM mass and the perturbative ($m_1/m_2 \ll 1$) behavior. Four level curves are shown for different ratios of bare masses $M_R = 1, 0.75, 0.5$ and 0.25 , together with the corresponding CLAP results from Andrade and Price (AP) [7], the reduced mass extension of the Davis et al. (DRPP) [11] perturbation calculation, and the Green's function integrations of Lousto and Price (LP) [12].

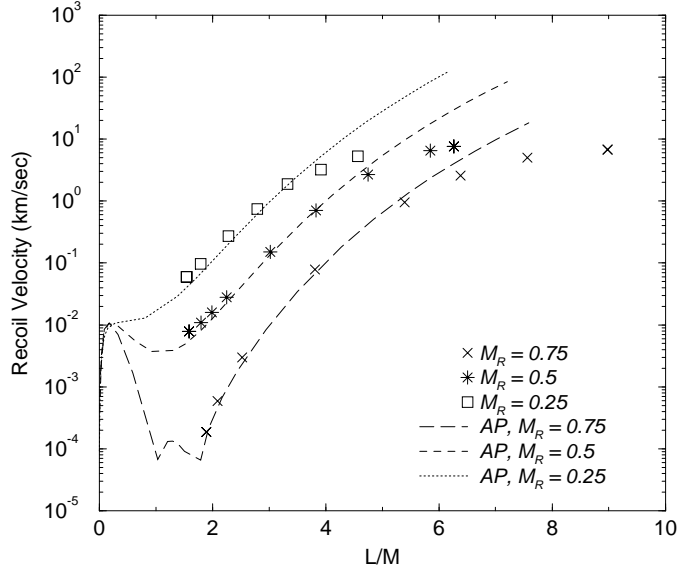


FIG. 3.

The absolute value of the recoil velocity is plotted together with the CLAP results from Andrade and Price (AP) [7] in units of km/sec for the three unequal mass level curves ($M_R = 0.75, 0.5$ and 0.25). Due to the difficulty in resolving the weaker high-order multipole components ($\ell \geq 3$) in the close separation limit, the numerical data is shown only for $(\mu_1, \mu_2) \gtrsim 0.5$, or roughly $L/M \gtrsim 2$. The results here (and in Fig. 2) are from simulations run at a grid resolution of 300×70 (radial \times angular) zones.